QUALITATIVE MATHEMATICAL MODELING FOR RESILIENCE ASSESSMENT AND MANAGEMENT.

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Abstract:

Since the 1970s resilience has been a lively discussed concept in ecology and environmental management. Resilience refers to "a measure of the persistence of systems and of their ability to absorb change and disturbance and still maintain the same relationships between populations or state variables". Despite important theoretical and empirical developments, the assessment of resilience of environmental systems has remained an unsolved challenge. Relevant variables and relations cannot always be immediately studied through exact equations or numerical methods. Variables usually differ in nature (e.g., natural, social), can be numerous, and might not be readily measured. The available data is commonly partial and not always quantitative. In this paper, assessing resilience of regimes of an environmental system will be based on the feasibility of representing regimes with loop models, which are developed with the qualitative mathematical modeling method of loop analysis. The importance of persistence in the definition of resilience calls attention to a mathematical treatment of stability in order to assess resilience. The present paper proposes that if a regime is resilient, the loop model representing the regime is stable. Correspondingly, if the regime is not resilient, the loop model representing the regime is not stable. In environmental management, promoting the persistence (resilience) of a desired regime, and the change of non desired ones, can be supported by analyzing what the influences (links) between the components of the regime are, or should be, so that the regime remains, or becomes, resilient and thus representable by a stable loop model. The approach introduced in the paper is explained with empirical evidences from three regimes observed in a wetland environment. It is shown that loop models can be obtained from scarce empirical data. Loop analysis allows alternative interpretations of the environmental system of interest, and addressing manifestations of non linearity.

Keywords: resilience; persistence; regime; stability; loop; feedback.

1. Introduction

Holling's (1973) foundational definition of resilience in ecology refers to "a measure of the persistence of systems and of their ability to absorb change and disturbance and still maintain the same relationships between populations or state variables". Assessing resilience of an environmental system is basically an exploration of relations in the system [13,18,41]. The core of such an exploration is the discussion of hypotheses about the interaction of the system components and analysis of what makes the environmental system persist [16,21,42-43]

The assessment of resilience has remained an unsolved challenge. Critical is the definition of a suitable methodological framework for articulating the diverse theoretical and empirical insights available in a particular resilience assessment [1,17,21,23,25,39].

Fundamental demands of such a framework include arranging or portraying the relations between the components (variables) considered relevant in the environmental system. However, these relations cannot always be immediately studied through exact equations or numerical methods (e.g., due to theoretical or practical constraints). It should be possible to address relations between components (variables) commonly different (e.g., natural, social), which might not be readily measured or whose available data is partial and not always quantitative.

These demands reflect the complexity of environmental systems, and suggest the use of qualitative mathematical modeling to help achieving a framework for resilience assessment [9-11,19,31,33-34,36-37]. The most influential literature in resilience research has also recognized that more quantification will not always increase understanding [21-23,42-43].

Assessing resilience is intimately related to addressing regime shifts. For a given territory, ecosystem or environmental system in general, a regime shift refers to the change between two fundamentally different statuses (named regimes) [25,42]. Regime shifts are commonly addressed regarding the change between well established efficient environmental management, and inefficient environmental management. For example, Kinzig et al. [25] explore potential regime shifts and future outcomes for the Causse Méjan region of France, where the observed decline of its native grasslands might undesirably transform the local culture and gastronomy.

The regimes preceding or resulting from a shift have different feedbacks and internal controls regarding their ecological, social, economic, cultural and environmental components [25]. Since regime shifts involve marked changes, they have been interpreted as manifestations of non linearity in environmental systems [1,21-22,42]. Promoting the desired status for a territory or environmental system can be interpreted as trying to promote the resilience (persistence) of desired regimes, and avoiding their shifts into non desired ones.

The present article proposes how to ascertain if a particular regime of an environmental system is resilient, and how management can use the assessment of resilience to promote the persistence or change of a regime. The methodological approach regards the feasibility of using qualitative mathematical models for representing a regime or situation under study. The approach is explained with empirical evidences from three regimes observed in a wetland environment.

2. Methodology
2.1 Characterization of three regimes observed in a wetland environment

On the south coast of Havana province in Cuba, three regimes were observed (Figure 1).
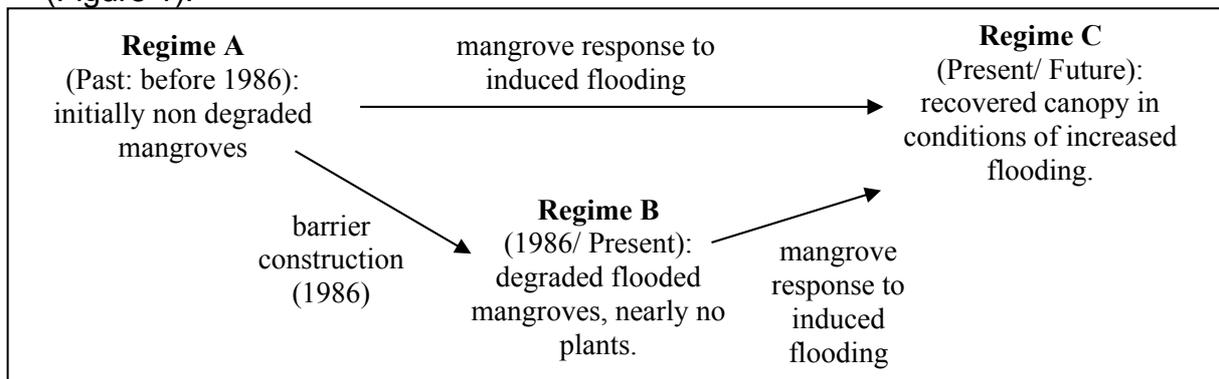
Fig. 1. Three regimes observed in a wetland environment with respect to a coastal freshwater barrier constructed in 1986.

The freshwater barrier was built across coastal mangroves and wetlands, in order to guarantee an adequate freshwater supply for agriculture and settlements located landward of the barrier. The barrier is 52 km long, and slows down the flow of freshwater into the sea by retaining water landward of the barrier. Besides achieving enhanced groundwater quality and quantity, the barrier caused mangrove dieback and negatively impacted forestry [6].

Regime A (Initially non degraded mangroves) refers to a mangrove forest without any major event leading to a permanent decrease in its vegetation cover. The construction of the barrier caused mangroves of Regime A to turn into Degraded flooded mangroves (Regime B). Regime A had persisted for years, as mangroves form the local natural vegetation [6]. Regime B had persisted since the barrier was built in 1986. In some sites, the barrier had not led to a total dieback in the mangrove cover but to an increase in more flood tolerant plant species (Regime C). The degraded flooded mangroves (Regime B) also show tendencies towards Regime C.

2.2. Assessing resilience with loop analysis

Assessing resilience of regimes of an environmental system will be based on the feasibility of representing regimes with loop models. A loop model is the type of model to be developed when using loop analysis, a qualitative mathematical modeling method adopted for ecology by [31]. The method was

developed to help dealing with complex systems as wholes [31,33], a common challenge in resilience assessment and environmental management. A system is considered a network of partly opposing and partly reinforcing processes, observable as changes in their intersections at specified variables [33].

The exploratory nature of a resilience assessment [1,7,21-22,25,42-43]   will be fulfilled with loop analysis. While developing and using a loop model, a fundamental step is to propose the variables considered relevant, and to discuss the interactions that represent how the variables influence each other. Therefore, gaining insight into the relations in the studied environmental system will be among the most fruitful outcomes.

The importance of persistence in the definition of resilience [21] calls attention to a mathematical treatment of stability in order to assess resilience. The present paper proposes that if a regime is resilient, the loop model representing the regime is stable. Correspondingly, if the regime is not resilient, the loop model representing the regime is not stable.

The stability criteria of loop analysis build on Lyapunov stability [9,33,38]. It can be understood as an ability to persist in the course of a sufficiently long time in spite of perturbations [40].

2.3 Basic aspects of loop analysis

Most of the applications of loop analysis have regarded dynamics of ecological communities and populations [5,8,10,14-15,28,32,35-36], one of the main original fields of study of Levins, who developed the method in ecology. More environment oriented applications have addressed cumulative effects (Lane, 1998), management options in riverine wetlands [4], and fisheries management [11].

Introductory or comprehensive overviews of the method are available [19,31,33,38,44], as well as explanations and discussions on some of its parts [2,3,9,26,27,29].

This paper uses the following requirements or tools of loop analysis: developing a loop model (also named signed graph), alternative loop models, stability of loop models, and response of variables to parameter change [31,33,38] Developing or proposing a loop model is the basic starting step in loop analysis, since all other tools will commonly use the loop model as a platform for analysis.

When developing or proposing a loop model, the influence of one variable on another variable is characterized by the sign of that influence; the method does not demand a complete equation to represent how a variable influences another variable (Figure 2).
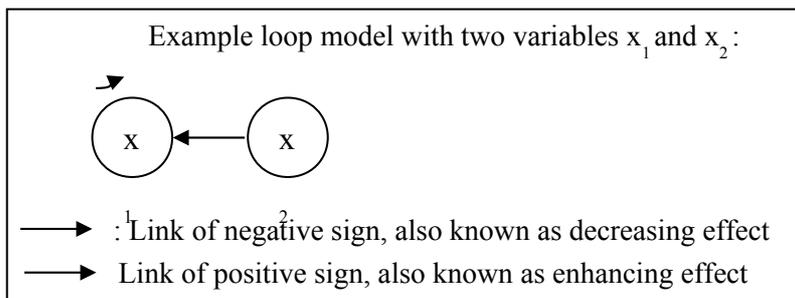
Fig. 2. Loop analysis: links (influences) between variables in a 2 variable loop model.

The sign (positive or negative) of the influence (link) can be proposed after empirical evidences or more theoretical propositions about the behavior of the variables involved in the link [8,30,36,38].

If a link is represented in a loop model, it indicates that if the variable source of the influence (where the link starts) changes, then the influenced variable (where the link ends as pictured by the arrow head) responds by changing faster or slower. In other words: its speed of variation (rate of change) is influenced. A loop model represents each influence (link) only qualitatively, with the sign of the influence. It corresponds to the sign of the differential $(\partial/\partial x_j)$ (dx_i/dt). For each variable in the loop model, the speed of variation (dx_i/dt) depends on all the variables in the loop model, i.e., $(dx_i/dt) = f_i (x_1, x_2, \ldots, x_n)$.

2.4 Loop models and resilience of the observed regimes

The loop models to be obtained for representing the three observed regimes (Figure 1) will include two variables. Taking only two variables will make clearer how loop analysis is proposed to assess resilience. The two variables are mangrove cover and infrastructure, because in the territory of interest, all processes can be interpreted as the intention of mangroves or infrastructure to persist [6]. Mangrove cover refers to the extension of mangrove vegetation as understood in vegetation science. Infrastructure refers to man made structures, which usually replace or influence vegetation during land cover and land use change; those structures include roads, freshwater barriers, buildings.

Since the persistence and hence resilience of the studied regimes is not neglected by their long actual (Regimes A and B) or potential durations (Regime C) (Figure 1), the notion of stability as the ability to persist in the course of a sufficiently long time in spite of perturbations [40] applies. Therefore, the three regimes (A, B and C) will be represented with stable loop models. Appendix 1 provides an example of how the stability of a loop model is assessed.

For the Regime A, the links to be proposed between the variables mangrove cover and infrastructure, and thus the loop model representing that regime, will be found by stepwise reconstruction of loop models [38]. The stepwise reconstruction will indicate the loop models that could have produced empirically observed tendencies of the variables (Table 1). This is supported by the loop analysis tool of parameter change, which indicates whether the values of variables increase, decrease, or remain unchanged in response to a change in the speed of variation of any of the variables [38].

Table 1. Evidence used in the stepwise reconstruction of loop models to find the 2 variable loop model(s) representing Regime A: decrease in mangrove cover and increase in infrastructure in response to a direct influence on infrastructure (construction of a freshwater barrier in 1986).

		Before barrier construction: 1970/ 1985	Barrier construction: Part of 1986.	After barrier construction: 1986/ present
Variable: mangrove cover (m)	Value	High (Mangroves as natural vegetation)	Decreasing	Low (Mangrove cover decreased)
	Speed of variation	About 0 (variable m about constant)	Negative (variable m started decreasing)	About 0 (variable m about constant)
Variable: infrastructure (i)	Value	Low	Increasing	Medium (Barrier already built)
	Speed of variation	About 0 (variable i about constant)	Positive (variable i started increasing)	About 0 (variable i about constant)

For the regimes B and C, finding their loop models will reflect an earlier explorative phase than for Regime A. These loop models will be found by proposing and discussing each particular link.

While finding the stable loop models that could represent the three regimes, the pool of candidates for stable 2 variable loop models will be conveniently restricted to those diagnosed as stable without needing to add more information on the links represented in the loop models [9,33,38].This will avoid discussing "ambiguity" in loop models, a relevant but at the same time different topic for which a solution was proposed by Dambacher [9] qualitative analysis of potential for stability.

3. Results
3.1. Assessing and managing resilience with the help of loop analysis
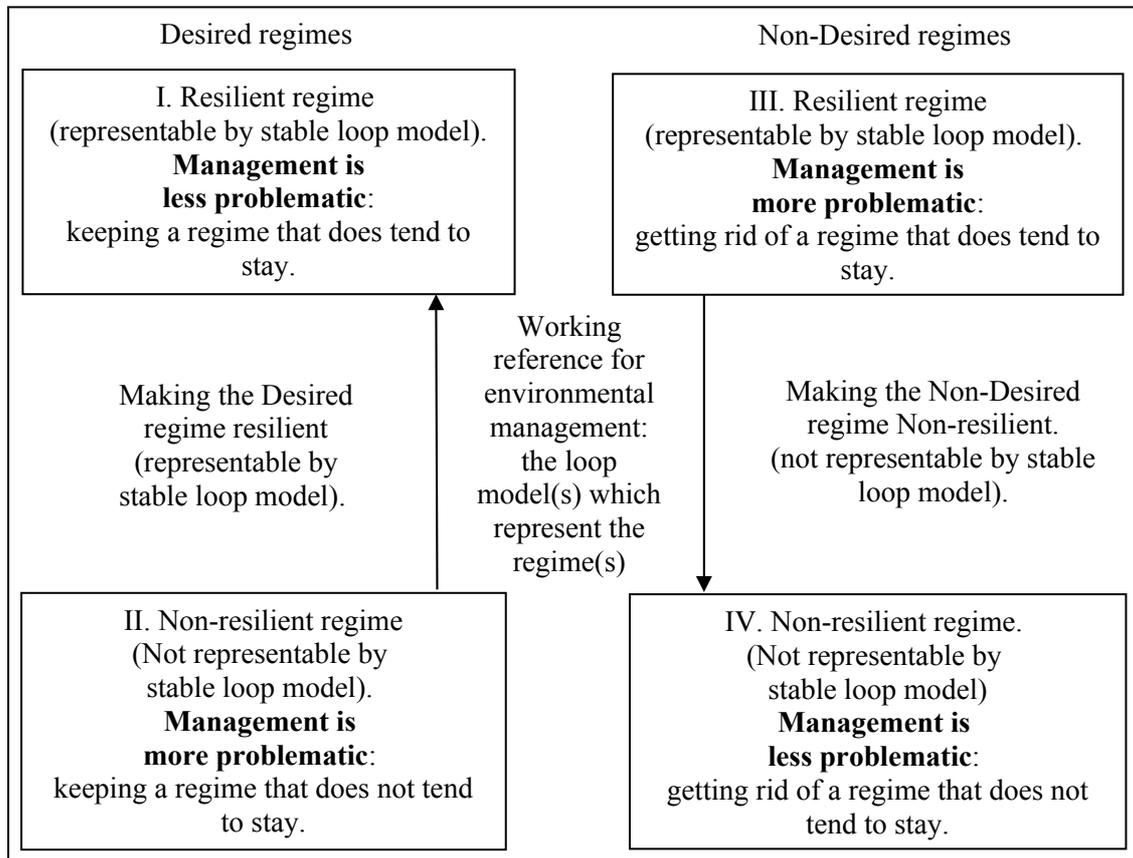
In environmental management, loop models can be a working reference to promote the persistence (resilience) of desired regimes (statuses) of an environmental system (Figure 3). Similarly, loop models can be a working reference to promote that non desired regimes (statuses) will not persist, i.e., to avoid the resilience of non desired regimes.

Having loop models as such a working reference means that environmental management would have to represent the regimes under analysis, the desired and/or non desired regimes, with loop models. Operationally, for each regime under analysis, environmental management would have to select which variables and links between variables should be included in the loop model(s) representing the regime.

The policy implication when dealing with a desired regime is that a desired regime will need to become, and remain, representable by stable loop model(s).

In that way, the desired regime would tend to persist in real life, i.e., to be resilient.

Similarly, the policy implication when dealing with a non desired regime is that a non desired regime shall not become, or remain, representable by stable loop model(s). In that way, the non desired regime would not tend to persist in real life, i.e., would not be resilient.

Desired regimes	Non-Desired regimes
I. Resilient regime (representable by stable loop model). **Management is less problematic**: keeping a regime that does tend to stay.	**III. Resilient regime** (representable by stable loop model). **Management is more problematic**: getting rid of a regime that does tend to stay.
Making the Desired regime resilient (representable by stable loop model).	Working reference for environmental management: the loop model(s) which represent the regime(s)	Making the Non-Desired regime Non-resilient. (not representable by stable loop model).
II. Non-resilient regime (Not representable by stable loop model). **Management is more problematic**: keeping a regime that does not tend to stay.	**IV. Non-resilient regime.** (Not representable by stable loop model) **Management is less problematic**: getting rid of a regime that does not tend to stay.

Fig. 3. Loop models as working reference for assessing and managing the resilience of desired and non desired regimes of an environmental system.

3.2. Loop models representing the regimes observed in a wetland environment

The stepwise reconstruction of loop models shows that models 1 and 2 can represent Regime A (Figure 4). Both models 1 and 2 could have produced the observed tendencies of mangrove cover and infrastructure before and after the construction of the barrier (Table 1). These two models differ in that Model 2 lacks the negative self feedback on infrastructure which is present in Model 1. For Regimes B and C (Figure 4), obtaining the links and thus the models is in itself a discussion (Table 2).
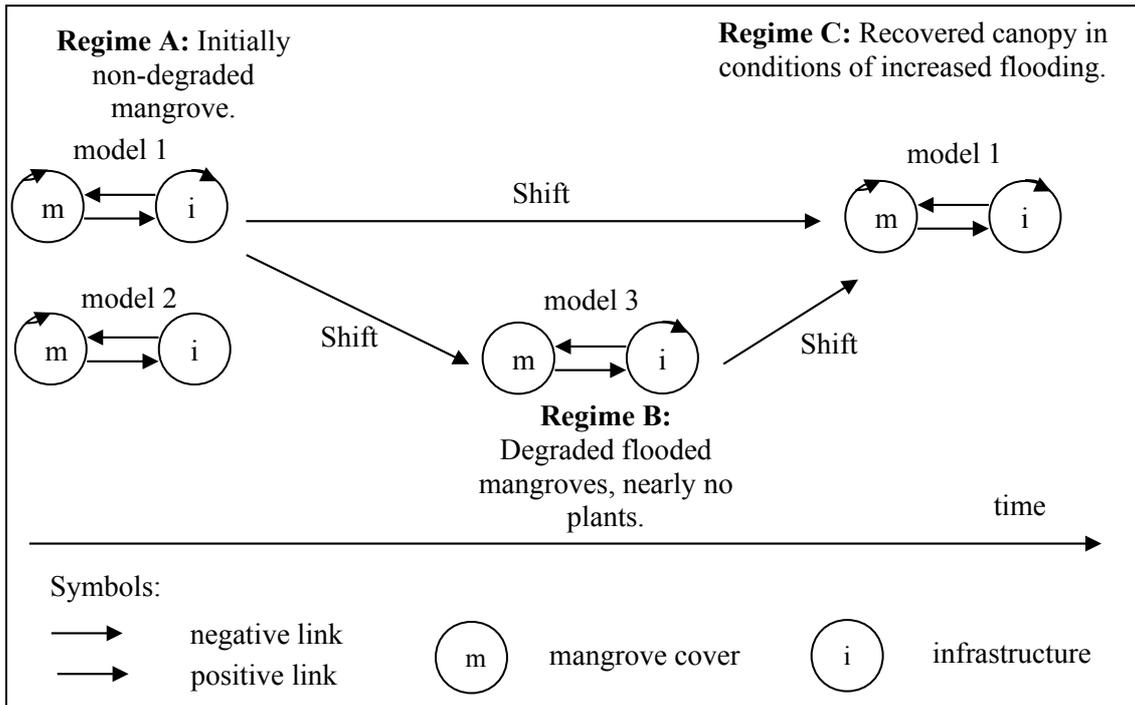
Fig. 4. Stable loop model(s) for the three regimes observed in a wetland environment.

Of the three regimes, Regime B (Degraded mangroves) is problematic for environmental management, not only because it is non desired [6], but also because it is resilient and so tends to persist (case III in Figure 3). Environmental management can try to get rid of that regime by implementing management actions that modify the qualitative structure of the environmental system. This means to modify the links between the variables mangrove cover and infrastructure so that such non desired regime will not remain representable by a stable loop model(s) and thus will stop being resilient and will not persist in real life.

The shifts between the observed regimes are potential manifestations of non linearity as existence of more than one regime for the same territory under study (Figure 5).
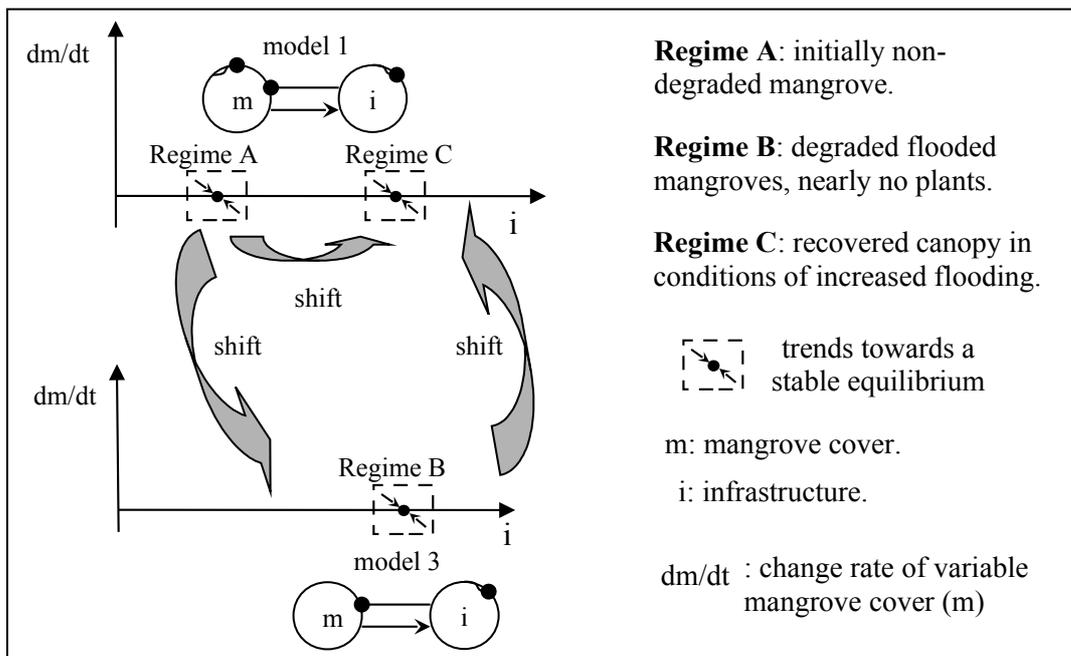
Fig. 5. Potential manifestation of non linearity as existence of more than one regime or stable equilibrium for the same territory under study.

4. Discussion

Resilience literature has explicitly supported the importance of qualitative understanding in resilience issues [21-23,42-43]. In one of Holling's most influential works [22], the author based such support on the same mathematical tools used by loop analysis, i.e., qualitative differential equations. He proposed a qualitative mathematical approach when addressing forces of change in ecosystems that have been subjected to management (e.g., suppression of forest fire).

Stable loop models as a working reference to address the persistence (resilience) of a regime of an environmental system (Figure 3) is conceptually equivalent to addressing such regime as tending to stay in a domain of attraction in the sense of Holling [21-22]. For a desired regime, it means managing the regime so that it becomes persistent (resilient), i.e., representable by a stable loop model. For a non desired regime, it means avoiding its persistence (resilience), i.e., avoiding that it is representable by a stable loop model. In this way, stability is taken as a reference for addressing change.

Interpreting the domain of attraction in the resilience literature with the local stability regions and fixed point concepts of the more mathematical literature [20,24, 38] has been implicit since Holling [21]. However, the author's stress on the need to assess change, and on differentiating stability from resilience [16,22-23,43], might have influenced the resilience literature to underexploit the mathematical concept of stability.
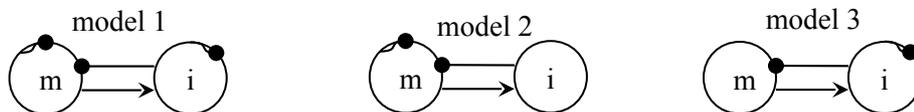
Stating that an environmental system can have different resilient regimes, each of them representable by a stable equilibrium, corresponds in the resilience literature to the welcomed idea of addressing environmental systems with a
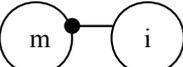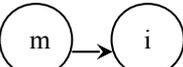
multiple equilibrium approach [7,22-23,25]. The changes from regime to regime are thus examples of changes from domain of attraction to domain of attraction [21,25]. Both the existence of more than one regime in an environmental system, and the multiple equilibrium approach of the resilience literature, has to do with manifestations of non linearity.

Loop analysis for addressing the resilience of three regimes in a wetland environment illustrates the relation between essential features of resilience assessment and management, and loop analysis. For example, having obtained two candidate models to represent Regime A (Figure 4) fits the need of resilience assessment and management to challenge different hypotheses about the interactions between the components of a regime. Independently of how much empirical evidence or theoretical knowledge is available, different hypotheses can emerge from controversial scientific views, or from stakeholders with different perceptions. Operationally, such hypotheses can be challenged by discussing the links that are in the loop models representing the regime [8, 30,31,33,35,38].

Discussing the links present in loop models is fundamental, not only when the links, and so the loop model(s), have been obtained via stepwise reconstruction as for the Regime A, but also in cases like Regimes B and C, whose loop models are obtained as a result of proposing and discussing each link (Table 2).

Table 2. Explanation of the links present in loop models representing three regimes observed in a wetland environment. Variables: mangrove cover (m) and infrastructure (i).

Link	Explanation
Present in models 1 and 2 (Regimes A, C) and model 3 (Regime B).	Negative influence of infrastructure (i) on mangrove cover (m): A change in the level of infrastructure causes a change of opposite sign to the change rate of mangrove cover. How it can happen: if a barrier is modified (increase in i) and depth of flooding water increases, then fewer mangrove seedlings can establish per year (decreased change rate of m).
Present in models 1 and 2 (Regimes A, C) and model 3 (Regime B).	Positive influence of mangrove cover (m) on infrastructure (i): A change in the level of mangrove cover causes a change of the same sign to the change rate of infrastructure. How it can happen: Successful mangrove restoration or naturally recolonized areas (increase in m) indicate that mangroves are healthier, and that they could stand more negative impacts. It can lead to decisions for building more infrastructures per year (increased change rate of i).

	Negative influence of mangrove cover (m) on mangrove cover (m): A change in the level of mangrove cover causes a change of opposite sign to the change rate of mangrove cover.
Present in models 1 and 2 (Regimes A, C).	How it can happen: When the aerial structure of plants increases, for instance when forest canopy recovers from gap opening (increase in m), then fewer saplings can establish per year in that more closed understory (decrease change rate of m).
Present in model 1 (Regimes A, C), and model 3 (Regime B).	Negative influence of infrastructure (i) on infrastructure (i): A change in the level of infrastructure causes a change of opposite sign to the change rate of infrastructure. How it can happen: Recently built infrastructure (increase in i) generates debate on its environmental impacts. Then, environmental regulations avoid placing new infrastructure, which results in fewer infrastructures built per year (decreased change rate of i).

Scarcity of environmental information usually challenges resilience assessment and management, and environmental management at large [34,37]. Loop analysis allowed handling the scarce information available for the regimes observed in the wetland environment.

When addressing Regime A, the tool stepwise reconstruction of loop models was restricted to rely on information of only one event influencing the variables of interest, i.e., the construction of a freshwater barrier in 1986, a direct influence on the variable infrastructure (Table 1). However, the tool could not indicate which of the two alternative models (1 and 2) was actually representing Regime A. The scarcity of information can be more acute, for instance if for that known event influencing the variables, the response of only one variable instead of the two variables would had been recorded. With less information entering the stepwise reconstruction of loop models, a higher number of loop models can result as candidates for representing Regime A. Having more abundant information is possible, for instance by knowing the response of the variables to other events different from the construction of the barrier. Such more abundant information can reduce the number of candidate models representing a regime. It requires a more detailed environmental history of a territory.

In the absence of more information on Regime A, deciding which of the candidate models (1 and 2) actually represents the regime had to rely on a discussion of the links present in the models. The only difference between the two models involves a negative self feedback on infrastructure, which is present in Model 2 but not in Model 1. Such negative self feedback means that the placement of infrastructure generates control over new placement of infrastructure. This control was present in the form of legal regulations and awareness of the need for proper use of natural resources [6]. Therefore, Model 1 was presumably representing Regime A. However, the negative self feedback on infrastructure could not have been very markedly strong, i.e., the initiation of the construction of the freshwater barrier (increase in infrastructure) did not prevent the construction of the barrier in the entire planned territory being completed. This suggests that the territory may actually have been represented

by the two alternative models, i.e., models 1 and 2. Interestingly, finding out which alternative model represents a specific regime is not always indispensable in loop analysis [30,33]; both models 1 and 2 would have helped to realize that the variable mangrove cover would decrease in the event of new infrastructure in form of a barrier, as was finally observed in reality.

With respect to the potential manifestations of nonlinearity in the wetland environment (Figure 5), in the path from Regime A to Regime C each regime corresponds to different stable equilibriums, which can be addressed by loop model 1. This case would be actual mathematical non linearity if the two equilibriums belonged to the same system of differential equations. A loop model specifies a system of differential equations only partially, through its signs, and not with full equations. Therefore, two different equilibriums addressed by the same loop model could still belong to different systems of differential equations.

Shifts between observed regimes in the wetland environment shows that a shift can certainly not be a manifestation of non linearity (Figure 5). This is illustrated by the shift from Regime A to Regime B, and the shift from Regime B to Regime C. Each regime involved in these shifts corresponds to different stable equilibriums addressed by different loop models (1 and 3), which represent different systems of qualitative differential equations. Therefore, these shifts are not manifestations of non linearity of the mathematical system (loop model) describing the territory, but a change of the mathematical system (loop model) for describing the territory.

A unified view on the mathematical and empirical application scope of loop analysis is needed. Justus [19] states that loop analysis is the qualitative counterpart of Lyapunov's indirect method, and as such it applies to a system in the local neighborhood of a point equilibrium. For enlarging the scope of qualitative stability analysis the author indicates the need for a qualitative version of Lyapunov's direct method in order to evaluate stability properties of systems outside the local neighborhood of equilibrium, which is not in the scope of loop analysis. Nonetheless, [19] recognizes situations in which the scope of loop analysis extends. The author refers to [12], who argues that [31-32] did not mathematically restrict loop analysis to qualitatively stable systems and proposes an extension of the method for tracking the behavior of stable systems.

Empirical evidences also indicate that the scope of loop analysis goes beyond stable loop models (i.e., stable equilibriums). Lane [29] should be reexamined, as the loop analysis tool of parameter change provides correct predictions in 165 out of 173 cases (94.8 %) with data from a natural plankton community. Similarly, Lane and Collins [27] should be reanalyzed; loop analysis allowed obtaining 201 correct predictions out of 211 cases (95.3%) in a land based mesocosm experiment for plankton communities.

In resilience research, clarifying the scope of loop analysis will necessarily involve addressing the trajectories of the environmental system between regimes (i.e., between equilibriums), for instance with the qualitative modeling approach (qualitative differential equations) of Petschel Held [37].

Methodological development will lead not only to define whether a regime is resilient or not. It shall be also possible to define how resilient a regime is. Support can come from Svirezhev's [40] mathematical interpretation of ecostability using an ecostability domain (which is similar to the concept of domain of attraction in the resilience literature), and Dambacher [9] qualitative analysis of potential for stability.

5. Concluding Remarks

Stable loop models developed in loop analysis, a qualitative mathematical modeling method, allow representing resilient regimes. In environmental management, promoting the persistence (resilience) of a desired regime, and the change of non desired ones, can be supported by analyzing what the influences (links) between the components of the regime are, or should be, so that the regime remains, or becomes, resilient and thus representable by a stable loop model.

The loop models for three regimes observed in a wetland environment show that these can be obtained from scarce empirical data. Loop analysis allows discussing alternative interpretations of the environmental system of interest, and addressing manifestations of non linearity. Opportunities should be taken to use quantitative methods for complementing the qualitative nature of the information to be included, and obtained, in qualitative methods.
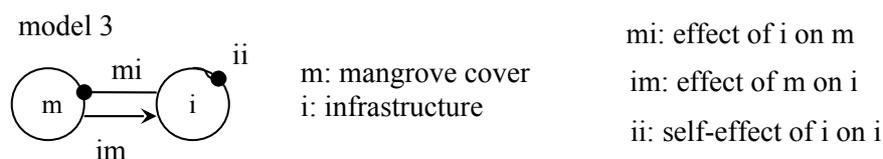
Acknowledgement

Appendix 1. Example assessing the stability of a loop model.

The two stability criteria for a loop model relate to Lyapunov stability applied to the zero solution of a system of differential equations. In loop analysis, those criteria are written as mathematical inequalities whose terms are the links of the loop model [9,38].

For the regimes observed in a wetland environment, loop model 3 (Figure 4) can represent Regime B. Model 3 is stable because it fulfills the two criteria for stability as shown here:

model 3

m: mangrove cover
i: infrastructure

mi: effect of i on m

im: effect of m on i

ii: self-effect of i on i

The first criterion for stability addresses the levels of feedbacks present in the loop model. A feedback is a way of grouping the links represented in a loop model. The number of levels to be addressed by this first criterion equals the

number of variables in the loop model. Thus, the first criterion for model 3 has to address two levels F_1 and F_2. For a loop model to fulfill the first criterion, feedbacks at all levels must be negative. So, $F_1<0$ and $F_2<0$ shall stand for model 3 to be stable. It is tested by applying the corresponding mathematical inequalities provided by loop analysis:

$F_1 = -(ii) < 0; F_1 < 0$

$F_2 = -(im)(mi) < 0; F_2 < 0$

Since both $F_1<0$ and $F_2<0$ stand, model 3 fulfills the first criterion for stability.

The second criterion for stability relates the negative feedback of long groups of links to the negative feedback of short groups of links. Verifying this criterion is demanded for loop models with more than 2 variables. In a 2 variable loop model as model 3 is, the fulfillment of the second criterion follows straight from the fulfillment of the first criterion as shown below.

For model 3 to fulfill the second criterion, applying the corresponding mathematical inequalities provided by loop analysis must yield $-F_1 > 0$. In criterion 1 it was found $F_1 < 0$; thus $-F_1 > 0$, model 3 fulfills the second criterion for stability.

Similar results ($F_1<0$, $F_2<0$; and $-F_1 > 0$) can be obtained for models 1 and 2 (Figure 4), so they are also stable loop models.

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